CKM Matrix from Leptoquark–Quark
Mixing in a String Model

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ABSTRACT

We investigate a scenario in which the CKM matrix arises from leptoquark–down quark mixing in a particular standard–like superstring model. We find that for some choices of F and D flat directions realistic quark mixing can be obtained without any phenomenological problems. This scenario predicts a symmetric (in absolute value) CKM matrix.

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1. Introduction

The origin of quark mixing is one of the fundamental questions in particle physics that the Standard Model does not answer. Extensions of the Standard Model such as models with supersymmetry or supergravity or grand unified theories are not improvements in this respect. All we are able to do with our present level of knowledge (or ignorance) is to parametrize quark mixing by the Cabibbo–Kobayashi–Maskawa (CKM) matrix in terms of three angles. Any theory such as superstrings [1] which claims to be the fundamental theory must be able to explain the origin and hopefully the magnitude of quark mixings. In the framework of standard–like superstring models [2], quark mixing was investigated in Refs. [3] and [4]. There, it was shown that nonzero off–diagonal elements in the up and down quark mass matrices require that some of the states $V_i, \bar{V}_i$ from the hidden sectors $b_i + 2\gamma$ get VEVs (due to the generational gauged $U(1)$ symmetries of these models). The source of quark mixing was identified to be the the VEVs of the hidden sector states $V_i, \bar{V}_i$. A specific set of scalar VEVs was shown to give correct order of magnitude quark mixing angles.

In this letter, we show that quark mixing can also arise as a result of the the presence of a pair of TeV scale leptoquarks which mix with the left and right–handed down quarks. Correct order of magnitude mixing angles can be obtained from proper amounts of leptoquark–down quark mixings which require specific scalar VEVs around M_{Pl}. This scenario also satisfies the constraints from the unitarity of the CKM matrix and flavor changing neutral currents (FCNC) due to Z exchange. We also investigate the related issue of the Nelson–Barr mechanism [5] as a solution to the strong CP problem. We find that in that case one cannot generate large enough quark mixing (and weak CP violation).

Similar ideas have been explored before either in a general framework [6] or in flipped $SU(5) \times U(1)$ superstring models [7] for $SU(2)_L$ singlet, heavy down quarks. In the former no concrete model was considered and the whole discussion was generic. In the latter, on the other hand, no estimate of quark mixing was

made due to the lack of calculational tools. In this letter, we consider a specific superstring model in which reliable estimates of all relevant terms can be made.

For concreteness we consider the generic standard–like superstring model of Ref. [8] which has leptoquarks in the massless string spectrum. The complete massless spectrum with the quantum numbers and the cubic superpotential was presented in Ref. [8] and will not be reviewed here. The notation of Ref. [8] is used throughout this letter.

2. Leptoquark–down quark mixing

In the massless $b_1 + b_2 + \alpha + \beta + (S)$ sector of the standard–like superstring model under consideration, there are two color triplet, electroweak singlet states, D_{45} and $\bar{D}_{45}$ [8]. Under $SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L$, D_{45} and $\bar{D}_{45}$ transform as $(3, 1, -1, 0)$ and $(\bar{3}, 1, 1, 0)$ respectively. In these models, $Q_Y = Q_C/3 + Q_L/2$ and $Q_{Z'} = Q_C - Q_L$ and therefore we find that $Q_Y(D_{45}) = Q_{EM}(D_{45}) = -1/3$ and $Q_{Z'}(D_{45}) = -1$ with $\bar{D}_{45}$ having the opposite charges. On the other hand, $Q_{B-L} = 2Q_C/3$ which is a gauge symmetry in these models. Thus, $Q_{B-L}(D_{45}) = -2/3$ and $Q_{B-L}(\bar{D}_{45}) = 2/3$. From the above quantum numbers we see that D_{45} and $\bar{D}_{45}$ are actually leptoquarks [9]. D_{45} and $\bar{D}_{45}$ are superfields and therefore there are two scalar and two fermionic leptoquarks in this model. Similar states have been identified in Calabi–Yau [1] and flipped $SU(5) \times U(1)$ string models [10] and were called vector–like down quarks.

The phenomenology of D_{45} and $\bar{D}_{45}$ including FCNC and baryon number (B) violating effects was investigated recently [9]. It was shown that FCNC constraints are easily satisfied due to the relatively large (i.e. $> TeV$) leptoquark masses and very small (i.e. $< 10^{-3}$) leptoquark Yukawa couplings. On the other hand, B violating effects may be dangerous since D_{45} and $\bar{D}_{45}$ may couple to diquarks and and lepton–quark pairs simultaneously. These induce large B violating operators unless some assumptions on the vanishing VEVs are made.

Leptoquark (from now on by leptoquarks or D_{45} and $\bar{D}_{45}$, we mean only the

fermionic ones since only they are relevant for our purposes) masses were discussed in detail in Ref. [9]. In general, one expects that D_{45} and $\bar{D}_{45}$ get large masses (of $O(10^{17}\ GeV)$) at the level of the cubic superpotential. Even if this is not the case D_{45} and $\bar{D}_{45}$ can get large masses from higher order (i.e. $N > 3$) terms in the superpotential and decouple from the low–energy spectrum. In Ref. [9] it was shown that all contributions to leptoquark masses (at $N = 3$ and $N = 5$) vanish due to the cubic level F constraints which must be imposed to preserve supersymmetry at M_{Pl}.

When hidden sector states are taken into account, there the are $N = 6$ terms $D_{45}\bar{D}_{45}T_2\bar{T}_2\Phi_{45}\Phi_2^+(\xi_1 + \xi_3)$ which may give large masses to D_{45} and $\bar{D}_{45}$. (Here $T_2, \bar{T}_2$ are $5, \bar{5}$ of the hidden $SU(5)_H$ gauge group.) If $\langle\Phi_2^+\rangle \neq 0$, then generically $\langle\Phi_2^+\rangle \sim M/10 \sim 10^{17}\ GeV$ and $\langle T_2\bar{T}_2\rangle \sim \Lambda_H^2$ where $\Lambda_H \sim 10^{14}\ GeV$ is the hidden $SU(5)_H$ condensation scale [4]. This gives $M_{D,\bar{D}} \sim 10^8\ GeV$. If $\langle\Phi_2^+\rangle = 0$, then the D_{45} $\bar{D}_{45}$ mass terms come from the SUSY breaking VEVs. The VEVs vanishing due to SUSY can become nonzero (and up to the TeV scale) once SUSY is broken. Therefore, when SUSY is broken, D_{45} and $\bar{D}_{45}$ get TeV scale masses from the cubic superpotential, i.e. from the term $W_{D,\bar{D}} = D_{45}\bar{D}_{45}\xi_3$ (since now $\langle\xi_3\rangle$ which vanished due to the supersymmetric F constraints is $\sim O(TeV)$). Thus, in this model, there are two fermionic leptoquarks with masses between $10^3\ GeV$ and $10^8\ GeV$ depending on the scalar VEVs.

The leptoquarks, D_{45} and $\bar{D}_{45}$, may mix with down–like quarks. In fact, there are nonrenormalizable terms which induce leptoquark mixing with right–handed down quarks of the form

$$d_3 D_{45} N_3 \Phi_{13} \Phi_3^+ \xi_i, \tag{1a}$$

$$d_2 D_{45} N_2 \Phi_2^- \xi_i, \tag{1b}$$

$$d_1 D_{45} N_1 \Phi_1^+ \xi_i, \tag{1c}$$

where ξ_i means $\xi_1 + \xi_2$. Similar mixing terms may also appear at higher orders but we neglect them since they are suppressed relative to those given above. $\langle N_i\rangle$

appears in nonrenormalizable terms which induce dimension four B and lepton number (L) violating operators [11]. Explicitly, the $N > 3$ terms which induce B and L violating terms in the superpotential are [11]

$$(u_3 d_3 + Q_3 L_3) d_2 N_2 \Phi_{45} \bar{\Phi}_2^-, \tag{2a}$$

$$(u_3 d_3 + Q_3 L_3) d_1 N_1 \Phi_{45} \Phi_1^+, \tag{2b}$$

$$u_3 d_2 d_2 N_3 \Phi_{45} \bar{\Phi}_2^- + u_3 d_1 d_1 N_3 \Phi_{45} \Phi_1^+, \tag{2c}$$

$$Q_3 L_1 d_3 N_1 \Phi_{45} \Phi_3^+ + Q_3 L_1 d_1 N_3 \Phi_{45} \Phi_3^+, \tag{2d}$$

$$Q_3 L_2 d_3 N_2 \Phi_{45} \bar{\Phi}_3^- + Q_3 L_2 d_2 N_3 \Phi_{45} \bar{\Phi}_3^-, \tag{2e}$$

In order to satisfy the constraints from the proton lifetime, the coefficients of the above B and L violating operators must be $< 10^{-13}$ (for sparticles with masses of $O(TeV)$) [12]. From this we get the constraint on the sneutrino VEVs, $\langle N_i \rangle \sim O(10^7 \ GeV)$ at most. There are no other phenomenological constraints on $\langle N_i \rangle$, therefore we conclude that $0 \leq \langle N_i \rangle < 10^7 \ GeV$.

In addition, there are leptoquark mixing terms with left–handed down quarks such as

$$Q_3 \bar{D}_{45} h_{45} H_{13} H_{23} V_3 \Phi_{45} \xi_i, \tag{3a}$$

$$Q_2 \bar{D}_{45} h_{45} H_{13} H_{23} V_2 \Phi_{45} \xi_i, \tag{3b}$$

$$Q_1 \bar{D}_{45} h_{45} H_{13} H_{23} V_1 \Phi_{45}. \tag{3c}$$

The only problem with these terms is the fact that supersymmetric F constraints at the cubic level of the superpotential require $\langle H_{13} \rangle = 0$ at the Planck scale (whereas H_{23} may get a nonvanishing VEV) [13]. On the other hand, it is plausible that higher order corrections to the superpotential modify the F constraints in such a way as to allow a large VEV for H_{13}. In the following we will assume this to be the case.

If the mixing terms in Eqs. (1) and (3) are nonzero, then we have a 4×4 down quark mass matrix, M_d of the form (in the basis (d, s, b, D_{45}))

$$M_d = \begin{pmatrix} m_d & 0 & 0 & m_3' \\ 0 & m_s & 0 & m_2' \\ 0 & 0 & m_b & m_1' \\ m_3 & m_2 & m_1 & M_D \end{pmatrix} \tag{4}$$

where we assume that there are no direct quark mixing terms. (This can be easily achieved by choosing the VEVs of $\bar{V}_i$ to be zero since direct mixing terms are proportional to $\langle V_i \bar{V}_j \rangle$. [3,4]) The case with only direct quark mixing terms was investigated previously [3,4]. It was found that, with a proper choice of scalar VEVs (F and D flat direction) a realistic CKM matrix can be obtained. We also take the up quark mass matrix, M_u, to be diagonal:

$$M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \tag{5}$$

There are no direct quark mixing terms in M_u either. In any case, it can be shown that the off–diagonal terms in M_u do not affect the CKM matrix by much. The CKM matrix arises mainly from the off–diagonal terms of M_d [3, 4]. m_i and m_i' which parametrize the leptoquark mixings with the right and left–handed down quarks respectively, are obtained directly from the mixing terms given by Eqs. (1) and (3).

The top mass is obtained at the cubic level from $u_1 Q_1 \bar{h}_1$ whereas the bottom, charm and strange masses are obtained, as usual, from $N = 5$ terms [11]

$$u_2 Q_2 (\bar{h}_{45} \Phi_{45} \bar{\Phi}_{23} + \bar{h}_1 \bar{\Phi}_i^+ \bar{\Phi}_i^-), \tag{6a}$$

$$d_1 Q_1 h_{45} \Phi_1^+ \xi_2, \tag{6b}$$

$$d_2 Q_2 h_{45} \bar{\Phi}_2^- \xi_1. \tag{6c}$$

5

The up and down quarks get masses from higher order terms [13]. Correct order of magnitude masses for all quarks can be obtained by a proper choice of scalar VEVs.

We now analyze the CKM matrix that arises from the above M_d under different assumptions about the mixing terms which are parametrized by m_i and m_i'.

3. The CKM matrix

The left–handed down quark mixing matrix which arises from the above M_d has been obtained before. To first order in the small parameters m_i/M_D and m_i'/M_D it is given by [6]

$$
V = \begin{pmatrix}
1 & \mu_{32}/m_s & \mu_{31}/m_b & -m_3'/M_D \\
-\mu_{32}^*/m_s & 1 & \alpha & -m_2'/M_D \\
-\mu_{31}^*/m_b & -\alpha^* & 1 & -m_2'/M_D \\
m_3'^*/M_D & m_2'^*/M_D & m_1'^*/M_D & 1
\end{pmatrix}
\tag{7}
$$

where $\mu_{ij} = m_i' m_j/M_D$ and $\alpha = [m_b/(m_b^2 - m_s^2)][\mu_{21} + (m_s/m_b)\mu_{12}^*]$. The terms of V are at most linear in the small parameters $\mu, m_i/M_D, m_i'/M_D$. As we will see later, higher order corrections to the elements of V are negligible unless these elments vanish. The right–handed down quark mixing matrix is given by $U = V(m_i \leftrightarrow m_i'^*)$. From Eq. (7) we see that $|V|$ is symmetric (up to corrections which will be discussed in the following) which is the most important prediction of this scenario for quark mixing.

The CKM matrix is the 3×3 block of V given above since there is no mixing in the up quark sector (or M_u). We want to see whether the elements of V can give us realistic quark mixing angles (we neglect all phases for our purposes or consider $|V|$). Now, the 3×3 CKM matrix becomes nonunitary because of the nonzero $4j$ and $i4$ elements in the 4×4 down quark mixing matrix, V. The strongest bounds on the magnitude of the new mixing terms parametrized by m_i and m_i' arise from the unitarity of the (3×3) CKM matrix V which imposes $|V_{uD}| < 0.07$ [14]

and from flavor changing Z currents [15] which imposes $|ReV_{id}^*V_{is}| < 2.4 \times 10^{-5}$, $i = u, c, t$. We would like to obtain realistic quark mixing without violating these phenomenological constraints for some choice of scalar VEVs (which appear in Eqs. (1) and (3)).

From the mixing matrix V given above, we see that the scalar VEVs must be such that $\mu_{32}/m_s \sim 0.2$, $\mu_{31}/m_b \sim 10^{-3}$ and $\alpha \sim 0.03$ to get the experimentally measured quark mixings. These values can be obtained by a suitable choice of the elements of M_d as follows. If $m_2/M_D \sim 1/5$, $m_3' \sim m_s$ and $m_1/m_b \sim 10$, then V_{us} and V_{ub} are of the correct order of magnitude. If in additon, we also have $m_2'/M_D \sim 3 \times 10^{-3}$ then we also get a realistic V_{cb}. We find that the unitarity constraint, $|V_{uD}| = m_3'/M_D < 0.07$ is easily satisfied by the above choice of values. In order to satisfy the constraint from FCNC, we must also satisfy $|ReV_{id}^*V_{is}| = m_1'm_2'm_3^2/M_D^2m_b^2 < 2.4 \times 10^{-5}$. This too can be easily satisfied by choosing $m_3/m_b \sim 2$ and $m_1'/M_D \sim 10^{-4}$.

Of course, all elements of M_d that we chose above result from some choice of scalar VEVs as dictated by the mixing terms given in Eqs. (1) and (3). In the following, we take $M_D \sim 1~TeV$ by imposing $\langle \Phi_2^+ \rangle = 0$. (1 $TeV << M_D < 10^5~TeV$ which is possible gives very small m_i/M_D and m_i'/M_D which in turn cannot produce appreciable quark mixing.) A choice of scalar VEVs which will produce the desired values for m_i' is

$$\langle H_{13}, H_{23} \rangle \sim M \qquad \text{and} \qquad \langle V_2, \Phi_{45}, \xi_1, \xi_2 \rangle \sim \frac{M}{10} \qquad (8)$$

with $\langle V_1 \rangle \sim \langle V_3 \rangle$ and $\langle V_2 \rangle \sim 20 \langle V_3 \rangle$. We take $\langle h_{45} \rangle \sim 150~GeV$ for our order of magnitude estimates. On the other hand, in order to obtain the desired values for m_i we can choose

$$\langle \Phi_{13}, \Phi_1^+, \Phi_2^-, \Phi_3^+ \rangle \sim \frac{M}{10} \qquad (9)$$

and $2\langle N_1 \rangle = \langle N_3 \rangle \sim TeV$ and $\langle N_2 \rangle \sim 20~TeV$.

There are a large number of F and D flat solutions (sets of scalar VEVs) which give the desired m_i and m_i' for this scenario. In general the F and D constraints will force other VEVs not required by the above mechanism to be nonzero too. For example, the D constraint for the hidden $SU(5)$ requires at least one $\bar{V}_i$ with nonzero VEV (which together with the V_i may produce direct quark mixing). Note that the required choice of scalar VEVs for this scenario is not the most natural one even though it is certainly possible. Generically scalar VEVs in these models are $\sim M/10$ whereas above we require relatively large VEVs (of $O(M)$) for H_{13}, H_{23} and a small VEV (of $O(M/10^2)$) for V_2.

From the mixing matrix V in Eq. (7) we see that all the off–diagonal elements vanish if either all m_i or m_i' are zero. This can happen if, for example, either $\langle N_i \rangle = 0$ or $\langle V_i \rangle = 0$. In that case one has to go to higher orders in the small parameters $\mu, m_i/M_D, m_i'/M_D$ to obtain the nonzero quark mixings. Higher order corrections to V when $m_i = 0$ are given by [16]

$$V_{ij} = \delta_{ij} + (1 - \delta_{ij}) \frac{m_i' m_j'}{M_D^2} \frac{m_i^2}{(-1)^{\delta_{ij}} m_j^2 - m_i^2} \tag{10a}$$

$$V_{i4} = -V_{4i} = -\frac{m_i'}{M_D}, \qquad V_{44} \sim 1 \tag{10b, c}$$

where we neglected all phases and m_i in the above formulas are now the down quark masses. This case is particularly interesting because it realizes the Nelson–Barr mechanism [5] for the solution of the strong CP problem. When all m_i in M_d given by Eq. (1) are zero, $Det(M_d)$ is real even if the m_i' (but not the diagonal elements, i.e. the quark masses) carry phases. Then if $\theta_{QCD} = 0$ for some reason, the θ angle does not get an additional contribution from the quark mass matrices since $\theta_{quark} = argDet(M_u M_d) = 0$. Due to the very small ratio $m_i'/M_D \sim 10^{-3-4}$ this scenario does not have any problems from supersymmetric processes either [17]. The hope is to obtain large enough weak CP violation (or quark mixing) solely from the phases of m_i' in Eq. (4). From the mixing terms in Eq. (10) for this case we find that this hope cannot be realized as was also noticed in Ref.

8

[17]. For example, the Cabibbo mixing, $V_{32} \sim 0.2$ can only be obtained from Eq. (10a) if $m'_i m'_j / M_D^2 \sim 4$ which means that we need $m'_i > M_D$ which is beyond the validity of our approximations. (We remind that all of the above formulas are expansions in the small parameters $\mu, m_i/M_D, m'_i/M_D << 1$). This result was also checked numerically and it was found that when $m'_i > M_D$ higher order corrections completely change the first order results.

The terms in Eq. (10) also give the first corrections to the elements of the mixing matrix V. Since $m'_i/M_D \sim 10^{-3-4}$ for our scalar VEVs, these corrections are at most about 10^{-3} times the matrix elements and therefore negligible compared to the lowest order terms in Eq. (7). From Eq. (10a) we see that, contrary to the elements of $|V|$, these small corrections are not symmetric. As a result, this scenario can only accomodate a CKM matrix whose absolute value is symmetric up to a few parts in a thousand.

4. Conclusions

To summarize, we have obtained correct order of magnitude quark mixing solely from the leptoquark–down quark mixing terms in the particular standard–like superstring model examined. There are a large number of F and D flat directions which produce this result (among the infinitely many which do not). The leptoquark masses $M_{D,\bar{D}}$ cannot be much larger than a few TeV for this scenario to work since otherwise $m_i/M_D, m'_i/M_D$ are too small to give appreciable quark mixing. Fortunately, such low masses are possible in this model. We found that when $\langle N_i \rangle = 0$ so that $m_i = 0$ in M_d one cannot get large enough quark mixing (or weak CP violation) and therefore the related Nelson–Barr mechanism is not realistic in this case.

The most important prediction of the scenario we discussed above is the symmetry of $|V|$, the absolute value of the mixing matrix. This can be easily seen from Eq. (7) for V (up to the first corrections given by Eq. (10) which are not symmetric). Since the first corrections to V are at most about a few parts in a

thousand, it will be very difficult to accomodate a nonsymmetric CKM matrix in this scenario. This property is model independent since it is a direct result of the form of M_d in Eq. (4) and does not depend on how m_i and m_i' arise.

In addition, there are model dependent predictions. For example, from Eqs. (1), (3) and (7) we find that

$$\frac{V_{ub}}{V_{cb}} = \frac{\langle V_3 \rangle}{\langle V_2 \rangle} \tag{11}$$

up to a few percent. (The corrections to this equality which are about a few percent mainly come from the approximation we use for $\alpha \sim m_2' m_1 / m_b$.) It would be intereseting to constrain the values of $\langle V_2, V_3 \rangle$ from other phenomenological phenomena and see if this relation holds. This, however, is a model dependent prediction and will only test this scenario in the framework of standard–like superstring models.

Acknowledgements

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